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METHOD OF CALCULATION OF A TURBULENT FLOW IN AN AXIAL GAP WITH A VARIABLE RADIUS BETWEEN A ROTATING DISK AND AN AXISYMMETRIC CASING

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An integral method for calculation of a turbulent flow in an axial gap between a rotating disk and an axisymmetric casing is developed with account for the small flow through the gap, the variation of the gap width over the radius, and the interaction with outer flows. Limitations of the mathematical models used by most researchers and ways of surmounting them are revealed. The method is confirmed by a comparison with known experimental data. The obtained computational integral parameters were used repeatedly to improve economy and reliability of industrial pumps, turbines, and compressors.

Introduction. The study of a turbulent flow in axial gaps between a rotating disk and a casing is of theoretical and practical importance. The economy and reliability of a large class of rotary machines – pumps, turbines, compressors, centrifuges – are determined by the parameters of this flow in many respects. Therefore, this type of flow was studied in many works. The most complete review and analysis are presented in [1, 2]. However, for the most part, the basic relations are obtained without a detailed evaluation of the adopted assumptions and also without account for essential special features of real turbomachines: radial leakage, variation of the gap width over the radius. In what follows, the assumptions adopted in the derivation of the basic equations are analyzed in detail and more general relations are formulated as applied to calculation of the most important parameters of turbomachines.

Basic Equations. For an analysis of an axisymmetric flow in an axial gap between a plane rotating disk and a casing (see Fig. 1) a cylindrical system of coordinates with the z axis coinciding with the axis of rotation and the origin of coordinates lying on the disk surface was used. In this case the equations of liquid motion can be presented in the following form (friction stresses in planes passing through the axis of rotation, which are considerably smaller for ordinary cases, are neglected):

$$v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\varphi^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{\rho}\frac{\partial \tau_r}{\partial z}, \tag{1}$$

$$v_r \frac{\partial v_\varphi}{\partial r} + v_z \frac{\partial v_\varphi}{\partial z} + \frac{v_r v_\varphi}{r} = \frac{1}{\rho}\frac{\partial \tau_\varphi}{\partial z}, \tag{2}$$

$$v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{\rho r}\frac{\partial (\tau_r r)}{\partial r}, \tag{3}$$

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0. \tag{4}$$

The integral equation

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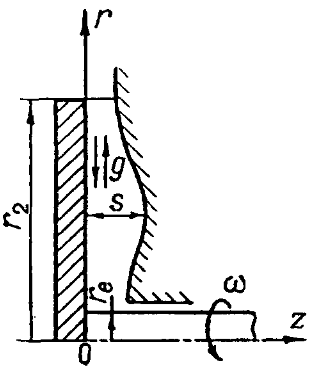


Fig. 1. Basic dimensions and coordinate system of the computation model.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\int_0^s v_r v_\varphi dz\right) = \frac{1}{r}\left(\tau_{\varphi s} - \tau_{\varphi 0}\right) \tag{5}$$

can be derived from formulas (2) and (4) using a familiar technique [2, 3, and others]. It should be noted that a further simplification of Eq. (5), made in a number of papers [1-3 and others], by factoring the mean value $\langle v_\varphi \rangle$ outside the integral sign according to the mean-value theorem, is incorrect for most cases that are of interest in practice, due to the alternating profile at relatively low radial flow rates. Formal factoring of $\langle v_\varphi \rangle$ outside the integral sign has led in some cases to "mean" values of this parameter that considerably exceed the physical value within the range from 0 to s and to erroneous results. That is why detailed information about the variation of the radial velocity in an axial gap rather than the integral value of the radial flow rate is required.

The relation between the velocity field and the pressure variation can be obtained by subtracting the continuity equation (4) multiplied by v_r from relation (1) and replacing the two terms in the left-hand side by the identical expression

$$-\frac{v_r^2}{r} - \frac{v_\varphi^2}{r} - v_r^2\frac{\partial}{\partial z}\left(\frac{v_z}{v_r}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{\rho}\frac{\partial \tau_r}{\partial z}. \tag{6}$$

We integrate this equation over the width of the axial gap with account for the vanishing of the velocities on the boundaries:

$$\int_0^s \left(\frac{v_r^2}{r} + \frac{v_\varphi^2}{r}\right) dz + \int_0^s v_r^2 \frac{\partial}{\partial z}\left(\frac{v_z}{v_r}\right) dz = \frac{1}{\rho}\int_0^s \frac{\partial p}{\partial r} dz - \frac{\tau_{rs} - \tau_{r0}}{\rho}. \tag{7}$$

We estimate the second term in the left-hand side of the equation. For this purpose we replace the function v_r^2 under the integral sign by its maximum value $v_{\max}^2$ within the integration range and obtain the following inequality:

$$\left|\int_0^s v_r^2 \frac{\partial}{\partial z}\left(\frac{v_z}{v_r}\right) dz\right| \le v_{r\max}^2 \left|\left.\frac{v_z}{v_r}\right|_s - \left.\frac{v_z}{v_r}\right|_0\right|. \tag{8}$$

We find the value $\left.\frac{v_z}{v_r}\right|_0$ for a power-law distribution of the radial component of the velocity (see below). If we eliminate the axial component of the velocity using the continuity equation, we can obtain the boundary value near the disk:

$$\left.\frac{v_z}{v_r}\right|_0 = \lim_{z\to 0}\frac{v_z}{v_r} = \lim_{z\to 0}\frac{v_z'}{v_r'} = \lim_{z\to 0}\frac{-z^{1/m}\left(\frac{v_{r0}}{r} + \frac{\partial v_{r0}}{\partial r}\right)}{z^{1/m-1}\, v_{r0}\,\frac{1}{m}} = 0. \tag{9}$$

The value of the same ratio near the casing is estimated similarly.

Thus, the right-hand side of relation (8) is equal to zero. Therefore, neglecting the variation in the pressure over the width of the axial gap, we can obtain the following equation for the relation between the pressure and the velocity field:

$$\int_0^s \left(\frac{v_r^2}{r} + \frac{v_\varphi^2}{r}\right) dz = \frac{1}{\rho}\frac{d(ps)}{dr} - \frac{\tau_{rs} - \tau_{r0}}{\rho}. \quad (10)$$

This equation can also be obtained in direct consideration of the balance of forces for an annular element. In the majority of works the last term in the right-hand side is neglected. This is admissible for relatively low radial flow rates through the axial gap, which are the most interesting in practice, but not in the general case. The left-hand side of relation (8) can be represented using the continuity equation (4) and the condition of zero velocities on the walls:

$$\left|\int_0^s v_r^2 \frac{\partial}{\partial z}\left(\frac{v_z}{v_r}\right) dz\right| = \left|\int_0^s \left[\frac{1}{r^2}\frac{\partial (v_r^2 r^2)}{\partial r} + \frac{\partial (v_r v_z)}{\partial z}\right] dz\right| = \left|\frac{1}{r^2}\frac{\partial}{\partial r}\int_0^s r^2 v_r^2 dz\right|. \quad (11)$$

Hence, allowing for the previous result, we obtain the following equation for determining the radial component of the velocity :

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \int_0^s v_r^2 dz\right) = 0. \quad (12)$$

It is close in form to the equality obtained directly from the continuity equation (4):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \int_0^s v_r dz\right) = 0. \quad (13)$$

As a result of integration of the latter relation we obtain an expression for determining the flow rate through the axial gap that will be used below:

$$g = 2\pi r \int_0^s v_r dz\rho. \quad (14)$$

Solution of the Equations and Determination of the Integral Parameters. Equations (5), (10), (12) are transformed to the following dimensionless form, with the parameters on the outer diameter of the disk or the axial gap being used as scales for leakage directed toward the axis of rotation, and the parameters on the inner diameter of the axial gap being used as scales for leakage away from the axis of rotation. The width of the axial gap varying over the radius serves as the scale of the axial coordinate:

$$\frac{\partial}{\partial R}\int_0^1 V_r^2 dZ + \left(\frac{2}{R} + \frac{d \ln S}{dR}\right)\int_0^1 V_r^2 dZ = 0, \quad (15)$$

$$\frac{\partial}{\partial R}\int_0^1 V_r V_\varphi dZ + \left(\frac{2}{R} + \frac{d \ln S}{dR}\right)\int_0^1 V_r V_\varphi dZ = \frac{T_{\varphi s} - T_{\varphi 0}}{S}, \quad (16)$$

$$\frac{d}{dR}(PS) = S\int_0^1 \frac{V_r^2 + V_\varphi^2}{R} dZ - (T_{rs} - T_{r0}). \quad (17)$$

At a constant width of the axial gap (S = const) the relations are simplified due to elimination of terms with factors involving logarithms. For the solution of Eqs. (15) and (16) sets of one-parameter distributions of the radial and circumferential components of the velocities formed by superposition of symmetric (transit) and skew-symmetric (shear) profiles of velocity were used. They also satisfy the conditions of liquid sticking to the wall, the power (with the exponent varying over the radius) law of velocity distribution directly near solid surfaces, and Eq. (14):

$$V_r = aZ^{1/m}(1-Z)^{1/m}\left(u_{20}RZ - \frac{1}{2}u_{20}R + \frac{G}{\alpha\, RB\left(1+\frac{1}{m}, 1+\frac{1}{m}\right)}\right), \tag{18}$$

$$V_\varphi = u_{20}R(1 - Z^{1/m}) + kZ^{1/m}(1-Z)^{1/m}. \tag{19}$$

These distributions are particular cases of more general relations of [4] that are also valid for a rotating casing. When $Z \to 0$, the profiles of the radial and relative circumferential components of the velocities and the profiles of the full relative velocities tend to the following power form:

$$V_r = Z^{1/m} a\left(\frac{G}{\alpha\, RB\left(1+\frac{1}{m}, 1+\frac{1}{m}\right)} - \frac{u_{20}R}{2}\right) = Z^{1/m} V_{r0}, \tag{20}$$

$$W_\varphi = V_\varphi - u_{20}R = -Z^{1/m}(u_{20}R - k) = Z^{1/m} W_{\varphi 0}, \tag{21}$$

$$W = \sqrt{W_\varphi^2 + V_r^2} = Z^{1/m}\sqrt{W_{\varphi 0}^2 + V_{r0}^2} = Z^{1/m} W_0. \tag{22}$$

Similar relations can be obtained for $Z \to 1$. To determine boundary friction stresses we use the Al'tshul' formula [5] for the coefficient of fluid friction, confirmed by familiar experiments on flows in tubes and boundary layers:

$$\lambda = (1.8 \log(\mathrm{Re}|7))^{-2}, \tag{23}$$

where $\mathrm{Re} = 2s\langle w\rangle/\nu$,

$$\langle w\rangle = v_2 \int_0^1 W_0 Z^{1/m} dZ = \frac{m W_0 v_2}{m+1} \tag{24}$$

is the mean-flow-rate velocity corresponding to a power profile. In the same paper an equation for the relation between the exponent of the profile and the coefficient of fluid friction is given:

$$m = \frac{1.28}{\sqrt{\lambda}} - 1. \tag{25}$$

These relations allow one to determine the dimensionless friction stresses on the disk surface, corresponding to the limiting velocity profiles, and their projections, proportional to the corresponding velocity components:

$$T_{r0} = \frac{\lambda}{8}\langle W\rangle^2 \frac{V_{r0}}{W_0} = \frac{\lambda}{8}\left(\frac{m}{m+1}\right)^2 V_{r0} W_0, \tag{26}$$

$$T_{\varphi 0} = \frac{\lambda}{8}\left(\frac{m}{m+1}\right)^2 W_{\varphi 0} W_0. \tag{27}$$

Similar relations can be obtained for the friction stresses on the casing (when $Z \to 1$). Substituting expression (18) in equality (16), we can derive an equation for the distribution of the parameter a and, consequently, of the radial component of the velocity in the axial gap:

$$\frac{da}{dR} = -\frac{2a}{R}\left[1 + \frac{d\ln S}{dR}\left(\frac{R}{4} - \frac{G^2(4+3m)}{a^2R^3u_{20}^2 m\mathrm{B}^2\left(1+\frac{1}{m}, 1+\frac{1}{m}\right)}\right)\right]. \tag{28}$$

As a result of solving this equation we can obtain the distribution of the parameter a:

$$a = \frac{a_2}{R^2}\sqrt{\left(\left|\frac{S_2}{S} + \frac{4(4+3m)G^2}{a^2u_{20}^2 m\mathrm{B}^2\left(1+\frac{1}{m}, 1+\frac{1}{m}\right)}\left(\frac{S}{S_2} - 1\right)\right|\right)}. \tag{29}$$

The dependence is substantially simplified at $S = \text{const}$:

$$a = \frac{a_2}{R^2}. \tag{30}$$

Substituting expressions (18), (19), (27) and a similar dependence for the friction stress on the casing into (16), we can obtain a differential equation for the distribution of the parameter k of the circumferential component of the velocity:

$$\frac{dk}{dR} = \frac{(T_{\varphi s} - T_{\varphi 0})R\mathrm{B}\left(1+\frac{1}{m}, 1+\frac{1}{m}\right)}{GS\mathrm{B}\left(1+\frac{2}{m}, 1+\frac{2}{m}\right)} + \frac{u_{20}^2R^2\mathrm{B}\left(1+\frac{1}{m}, 1+\frac{1}{m}\right)\mathrm{B}\left(1+\frac{1}{m}, 1+\frac{2}{m}\right)}{2G(3+2m)\mathrm{B}\left(1+\frac{2}{m}, 1+\frac{2}{m}\right)} \times$$

$$\times\, 2a\left[1 + \frac{d\ln S}{dR}\left(\frac{R}{4} + \frac{G^2(4+3m)}{a^2R^3u_{20}^2 m\,\mathrm{B}^2\left(1+\frac{1}{m}, 1+\frac{1}{m}\right)}\right)\right] -$$

$$-\,2u_{20}\frac{\mathrm{B}\left(1+\frac{1}{m}, 1+\frac{1}{m}\right)}{\mathrm{B}\left(1+\frac{2}{m}, 1+\frac{2}{m}\right)} + 2u_{20}\frac{\mathrm{B}\left(1+\frac{1}{m}, 1+\frac{2}{m}\right)}{\mathrm{B}\left(1+\frac{2}{m}, 1+\frac{2}{m}\right)} - \frac{k}{R}. \tag{31}$$

At $S = \text{const}$ the equation is considerably simplified:

$$\frac{dk}{dR} = \frac{(T_{\varphi s} - T_{\varphi 0})R\mathrm{B}\left(1+\frac{1}{m}, 1+\frac{1}{m}\right)}{GS\mathrm{B}\left(1+\frac{2}{m}, 1+\frac{2}{m}\right)} + \frac{u_{20}^2\mathrm{B}\left(1+\frac{1}{m}, 1+\frac{1}{m}\right)\mathrm{B}\left(1+\frac{1}{m}, 1+\frac{2}{m}\right)a_2}{G(3+2m)\mathrm{B}\left(1+\frac{2}{m}, 1+\frac{2}{m}\right)} -$$

$$-\,2u_{20}\frac{\mathrm{B}\left(1+\frac{1}{m}, 1+\frac{1}{m}\right) - \mathrm{B}\left(1+\frac{1}{m}, 1+\frac{2}{m}\right)}{\mathrm{B}\left(1+\frac{2}{m}, 1+\frac{2}{m}\right)} - \frac{k}{R}. \tag{32}$$

The boundary values of a and k at $R = 1$, necessary for the solution of the equations, are determined by outer flows with respect to the axial gap. Unfortunately, measurements of velocity profiles in the region of conjugation of the flow in the axial gap with outer flows of real turbomachines are virtually absent. Therefore, boundary values of the parameters were evaluated approximately or by results of comparison of experimental and calculated distributions of pressure. The latter technique was used to obtain an empirical estimate of the parameter a of the boundary profile of the radial component of the velocity in leakage through the axial gap directed toward the axis:

$$a_2 = -0.1 . \tag{33}$$

The boundary value of k of the profile of the circumferential component of the velocity with leakage directed toward the axis of rotation can be found approximately from the equation of momentum conservation in transformation of the near-wall power profile of the circumferential flow at the outlet from the working wheel in an infinitely thin layer to the profile (19):

$$\int_0^1 V_u (1 - Z)^{1/m} a_2 Z^{1/m} (1 - Z)^{1/m} \left(u_{20} Z - \frac{1}{2} u_{20} + \frac{G}{a_2 \mathrm{B}\left(1 + \frac{1}{m}, 1 + \frac{1}{m}\right)} \right) dZ =$$

$$= \int_0^1 \left[u_{20} (1 - Z^{1/m}) + k_2 Z^{1/m} (1 - Z)^{1/m} \right] a_2 Z^{1/m} (1 - Z)^{1/m} \times$$

$$\times \left(u_{20} Z - \frac{1}{2} u_{20} + \frac{G}{a_2 \mathrm{B}\left(1 + \frac{1}{m}, 1 + \frac{1}{m}\right)} \right) dZ . \tag{34}$$

From the latter equation we determine the boundary value of the parameter k_2:

$$k_2 = \frac{u_{20}}{\mathrm{B}\left(1 + \frac{2}{m}, 1 + \frac{2}{m}\right)} \times \left[\left(1 - \frac{V_u}{u_{20}}\right) \frac{a_2 u_{20} \mathrm{B}\left(1 + \frac{1}{m}, 1 + \frac{1}{m}\right) \mathrm{B}\left(1 + \frac{1}{m}, 1 + \frac{2}{m}\right)}{2G (3 + 2m)} + \right.$$

$$\left. + \mathrm{B}\left(1 + \frac{1}{m}, 1 + \frac{2}{m}\right) \left(1 + \frac{V_u}{u_{20}}\right) - \mathrm{B}\left(1 + \frac{1}{m}, 1 + \frac{1}{m}\right) \right] . \tag{35}$$

Similarly, for the flow from the axis of rotation, issuing from the slit sealing of the shaft to the axial gap, the circumferential velocity of whose surface is u_e, we can find approximate values of the boundary parameters:

$$a_e = -0.4 , \tag{36}$$

$$k_e = R_e . \tag{37}$$

Thus, using Eqs. (29) and (31) and the values of the parameters on the boundary of the axial gap between the rotating disk and the casing, at the given leakage we calculate the distributions of the radial and circumferential components of the liquid velocity. Integrating Eq. (17), we can obtain a formula for calculating the pressure distribution in the axial gap from the known velocity field:

$$P_2 - P = \frac{1}{S} \int_R^1 \frac{S}{R} \left(\int_0^1 (V_r^2 + V_\varphi^2)\, dZ \right) dR - \frac{1}{S} \int_R^1 (T_{rs} - T_{r0})\, dR . \tag{38}$$

In most cases of relatively small leakage the last term can be neglected. Assigning a number of values of the leakage, we can find the corresponding pressure drops in the axial gap. The obtained hydraulic characteristic of the axial gap makes it possible to considerably refine losses to leakage in a turbomachine. Then, such important

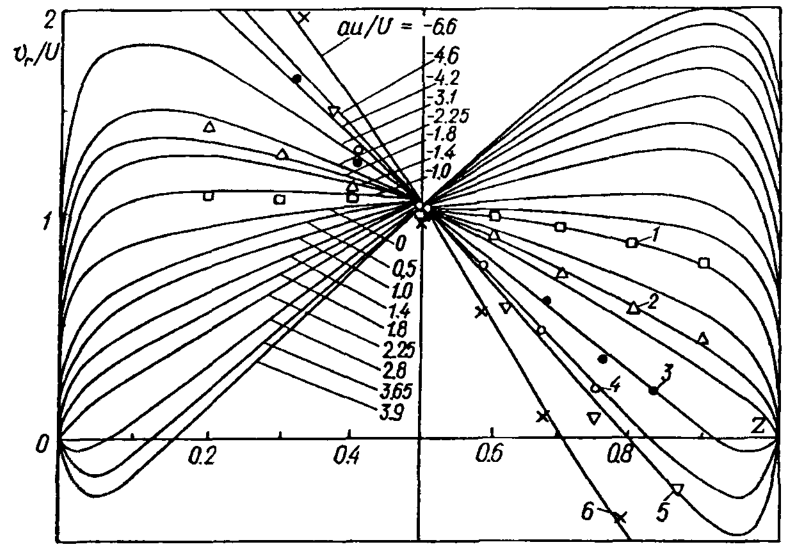


Fig. 2. Distribution of the radial component of the velocity (curves, calculation at $S = 0.03$, $m = 7$; points, experiment at $R = 0.7-1$; $S = 0.03-0.04$; $R_e = 0.1-0.2$): 1) $G_R = 0.137$ [6]; 2) 0.0893 [6]; 3) 0.035 [7]; 4) 0.029 [7]; 5) 0.0207 [7]; 6) 0.015 [8].

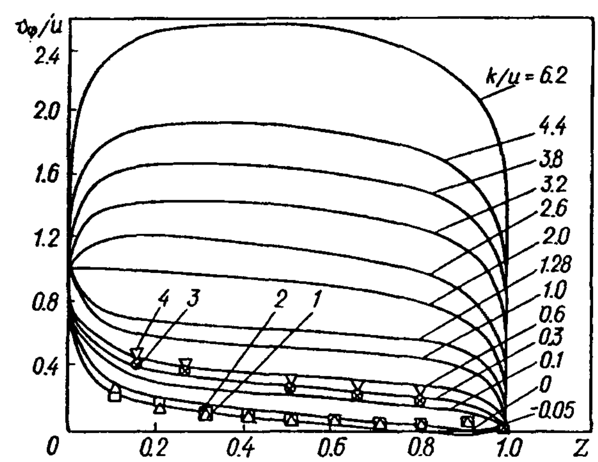


Fig. 3. Distribution of the circumferential component of the velocity (curves, calculation at $S = 0.03$, $m = 7$; points, experiment at $R = 0.7-1$; $S = 0.02-0.04$; $R_e = 0.1-0.2$): 1) $G_R = 0.137$ [6]; 2) 0.0893 [6]; 3) 0.044 [8]; 4) 0.022 [8].

integral parameters for turbomachines as the axial force acting on the disks and greatly affecting the reliability of the operation of the turbomachine supports

$$A = 2\pi \int_{r_e}^{r_2} prdr \tag{39}$$

and power losses to disk friction

$$N = c_{\mathrm{fd}} \rho r_2^5 \omega^3 \tag{40}$$

are determined. In the latter expression the coefficient of the moment of disk resistance is calculated from the known velocity field using dependence (27):

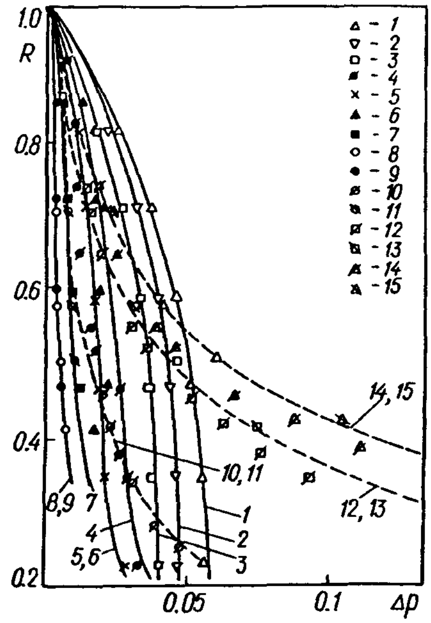


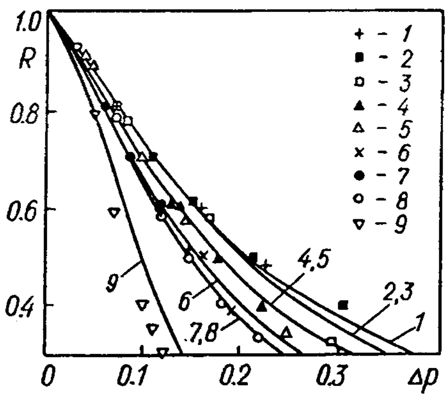
Fig. 4. Pressure distribution in leakage from the axis of rotation (curves, calculation at $S = 0.016-0.15$; $R_e = 0.1-0.25$; points, experiment): 1) $G_R = 0.001$ [8]; 2) 0.0015 [8]; 3) 0.002 [8]; 4) 0.003 [8]; 5) 0.0041 [8]; 6) 0.0044 [9]; 7) 0.0133 [9]; 8) 0.019 [6]; 9) 0.022 [9]; 10) 0.075 [6]; 11) 0.075 [10]; 12) 0.166 [10]; 13) 0.175 [6]; 14) 0.21 [10]; 15) 0.22 [6].

$$c_{fd} = \frac{2\pi}{u_{20}^2 R_2^3} \int_{R_e}^{R_2} T_{\varphi 0} R^2 dR . \tag{41}$$

It should be noted that for low-speed turbomachines with relatively narrow working channels these losses comprise an important part of all losses.

Comparison of Calculations with Experiments and Analysis of the Results. A computer program for a numerical solution of the differential equation and calculation of the integral parameters was developed on the basis of the relations derived. The results of calculations were compared with experimental data obtained in studies of, unfortunately, only an axial gap with a constant width. Figures 2 and 3 show good agreement of experimental and calculated profiles of the radial and circumferential components of the velocities for leakage from the axis, which, for convenience of comparison, are reduced to parameters different from those used in the computational technique. The alternating character of the profile of the radial component of the velocity in small leakages and its levelling in large leakages are typical. Measurements of the static pressure exerted on the wall of the axial gap are more numerous and reliable. Figure 4 shows calculated and experimental pressure distributions in leakage from the axis of rotation. As the leakage increases, the pressure curve first straightens due to transfer of weakly twisted liquid from the axis to the periphery and a general decrease in the level of the velocities in the axial gap (curves 7, 8, 9). However, with a further increase in the leakage the effect of the circumferential component of the velocity decreases and a flow with an elevated pressure, similar to that in a plane radial diverging segment, is realized near the axis. This leads to an alteration of the sign of the curvature of the pressure curve (curves 4, 5, 6). Then this flow covers the entire axial gap (curves 12, 13, 14, 15). It should be noted that the best convergence of the calculation and the experiment corresponded to an initial value of the parameter $a = -0.4$.

With a radial leakage directed toward the axis of rotation (Fig. 5 for relatively wide axial gaps, Fig. 6 for narrow axial gaps) in both experiments and calculations, which are in a good agreement with them, the decrease in pressure becomes stronger with increase in leakage (in absolute value). This occurs due to transfer of twisted liquid to the axis and an increase in the circumferential component of the velocity, as in the flow near a vortex sink. It should be noted that the curvature of the pressure curve has a different sign compared to the parabolic



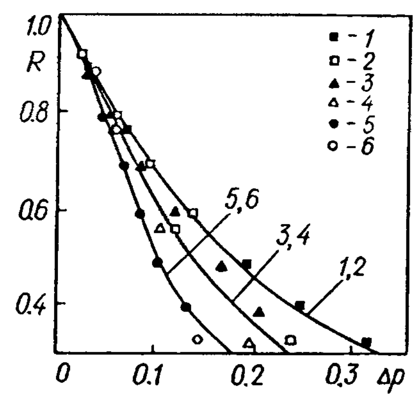


Fig. 5. Pressure distribution in leakage toward the axis of rotation in wide axial gaps (curves, calculation at S = 0.05–0.07; R_e = 0.3; points, experiment): 1) G_R = −0.012 and S = 0.05 [11]; 2) −0.0095 and 0.07 [12]; 3) −0.0095 and 0.07 [3]; 4) −0.0068 and 0.07 [12]; 5) −0.0068 and 0.07 [3]; 6) −0.0055 and 0.05 [11]; 7) −0.0048 and 0.07 [12]; 8) −0.0048 and 0.07 [3]; 9) −0.0011 and 0.06 [13].

Fig. 6. Pressure distribution in leakage toward the axis of rotation in narrow axial gaps (curves, calculation at S = 0.0173, R_e = 0.3; points, experiment): 1) G_R = −0.0381 [12]; 2) −0.0381 [4]; 3) −0.0272 [12]; 4) −0.0272 [4]; 5) −0.0167 [12]; 6) −0.0167 [4].

curve corresponding to liquid rotation according to the law of a solid body in the absence of leakage. Calculations and experiments showed that for any direction of leakage that has a large absolute value, the effect of the leakage decreases with increase in the width of the axial gap due to a decrease in the radial component of the velocity.

Conclusion. The integral method suggested for calculation of a turbulent flow in the axial gap between a rotating disk and a casing allows one to improve the accuracy of calculations and to take into account the most important special features of real machines: the values of leakage through the axial gap, variations of the axial gap width over the radius, effects of boundary flows. The developed interactive program complex, the main part of which was developed on the basis of this method, considerably increased the accuracy of calculations of axial forces affecting the rotor, leakages and associated volumetric losses, and losses to disk friction in the main types of centrifugal pumps. The method developed can be used for calculation of axial forces acting on disks of turbines and cooling systems and disk losses in turbines and centrifuges. The method can be generalized to flows between a rough rotating disk and a casing and to a compressible-fluid flow.

NOTATION

A, axial force acting on the disk; a, parameter of the profile of the radial component of the liquid velocity; B(..., ...), beta-function; c_{fd}, coefficient of disk friction; g, radial mass flow rate through the axial gap; $G = g/(2\pi\rho r_2 s_2 v_2)$, $G_R = g/(2\pi\rho r_2^2 s_2\omega)$, dimensionless radial flow rates; $R = r/r_2$, relative radius; k, parameter of the profile of the circumferential velocity of the liquid; $1/m$, exponent of the power distribution of the velocity; N, power losses to disk friction; $P = p/(\rho v_2^2)$, dimensionless pressure; p, pressure; Re, Reynolds number; s, axial distance between the disk and the casing; $S = s/r_2$, dimensionless axial distance; U, mean radial velocity in the cross section of the axial gap; u_{20}, dimensionless circumferential velocity on the outer diameter of the disk; v, absolute velocity of the liquid; $V = v/v_2$, dimensionless absolute velocity; $v_2 = (U_2^2 + (0.5\omega r_2)^2)^{1/2}$, velocity scale; w, relative velocity of the liquid; $W = w/w_2$, dimensionless relative velocity of the liquid; z, axial distance reckoned from the disk; $Z = z/s$, dimensionless axial distance; λ, coefficient of hydraulic friction; ρ, liquid density; τ, friction stress; T, dimensionless friction stress between the liquid and a solid surface; ω, angular velocity of the disk; $\Delta p = (p_2 - p)/(\rho\omega^2 r_2^2)$, dimensionless pressure drop. Subscripts: 0, on the disk surface; max, maximum value; r, radial component; s, on the casing surface; u, outer flow with respect to the axial gap; z, axial component; φ, circumferential component; 2, outer or initial radius of the axial gap; e, inner radius of the axial gap; $\langle\ \rangle$, mean-flow-rate value.

REFERENCES

1. T. Lingelbach and E. Wiederuh, Fortschritt-Berichte der VDI-Zeitschrift, 7, No. 154, 1-67 (1989).
2. W. Jedral, Metody obliczania sil wzdluznuch i ukladow odciazajacich w pompach wirowych (Methods of Calculation of Axial Thurst and Balancing Devices in Centrifugal Pumps), Warzawa (1988).
3. L. A. Dorfman, Hydraulic Resistance and Heat Transfer of Rotating Bodies [in Russian], Moscow (1960).
4. A. S. Baibikov and V. K. Karakhan'yan, Hydrodynamics of Auxiliary Ducts of Vane-Type Machines [in Russian], Moscow (1982).
5. A. D. Al'tshul', Hydraulic Losses to Friction in Pipelines [in Russian], Moscow (1963).
6. F. J. Bayley and J. M. Owen, Aeronautical Quarterly, No. 9, 333-354 (1969).
7. M. I. Tsaplin, Énergomashinostroenie, No. 8, 15-18 (1967).
8. D. Altmann, Pumpen und Verdichter Information, No. 2, 21-32, 52-55 (1973).
9. O. A. Verbitskaya, Pressure Distribution in Side Pockets of Centrifugal Pumps with Account for Leakages [in Russian], Moscow (1957).
10. M. Leidet, Computer Rendus des Seances de L'Academie de Sciences, 277, No. 18, 919-922 (1973).
11. J. Kurokava and T. Toyokura, in: Int. Conf. on Pump and Turbine Design and Development, East Kilbridge (1976), pp. 315-331.
12. A. S. Baibikov, V. B. Shnepp, and S. S. Evgen'ev, Kompressor. Kholod. Mashinostroenie, No. 4, 26-29 (1970).
13. Y. Senoo and H. Hayami, Trans. ASME, I, No. 2, 192-198 (1975).